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Uncertainties in transient heat transfer measurements with liquid crystal

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Abstract

Thermochromic liquid crystal (TLC) is used extensively as an experimental tool to determine heat transfer coefficients. In a transient experiment, if the time at which the TLC changes colour is known, then h, the heat transfer coefficient, can be found from the solution of the so-called semi-infinite-plate problem. If T_{aw}, the adiabatic-wall temperature, is unknown, then two narrow-band TLCs can be used to determine both h and T_{aw}. In this paper, an uncertainty analysis is used to calculate P_h, the uncertainty in h, and $P_{T_{aw}}$, the uncertainty in T_{aw} (when T_{aw} is unknown), in terms of the random uncertainties in the measured temperatures. Computed values, obtained using a Monte Carlo method, are in good agreement with the uncertainties obtained from the analysis. It is also shown how the uncertainties P_h and $P_{T_{aw}}$ can be minimised by selecting the appropriate ranges of TLC. Conversely, a poor choice of TLC can result in large values of these uncertainties. © 2002 Published by Elsevier Science Inc.

Keywords: Liquid crystal; Uncertainties; Transient heat transfer

1. Introduction

Thermochromic liquid crystal (TLC) is now widely used to determine heat transfer coefficients: see, for example, Ireland and Jones (1985), Jones and Hippensteele (1988), Kasagi et al. (1989), Camci et al. (1991) and Baughn (1995). TLC has the property that its colour changes over a defined range of temperatures: for narrow-band TLC, the transitional temperature range is around 1 °C, and the typical uncertainty in measuring this temperature is approximately 0.1 °C; for wide-band TLC, the range can be 10 °C or greater, and the uncertainty is correspondingly larger than for the narrow-band.

The heat transfer coefficient, h, is defined here as

$$h = \frac{q_w}{T_w - T_{aw}}, \tag{1.1}$$

where q_w is the heat flux from the surface to the fluid, T_w is the surface, or wall, temperature, and T_{aw} is the adiabatic-wall temperature. For some cases, T_{aw} is known or assumed. For example, for flow over a flat plate, it is often assumed that T_{aw} is equal to the total temperature of the free-stream. When T_{aw} is unknown, it can be found experimentally, as discussed below.

Heat transfer experiments using TLC are either steady-state or transient, and only the latter case is considered here. For example, a test piece, made from a poor thermal conductor such as acrylic plastic, is coated with narrow-band TLC, which has been calibrated against temperature. In a typical experiment in a wind tunnel, the test piece is subjected to a step-change in air temperature. As soon as the TLC reaches its "transition temperature", it will change colour: the higher the heat transfer coefficient, the shorter the time to reach transition. A video-recording of the surface would reveal a coloured contour, related to the transition temperature, that moves in time from regions of high to low h. Knowing the initial temperature, the transition temperature and the time required to reach transition, contours of h can then be calculated from the recorded contours of temperature.

The usual way of calculating h for the transient experiment is from the step-change solution of the

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Nomenclature

c	specific heat of plate
d	thickness of plate
$f(\alpha)$	solution of semi-infinite-plate problem
h	heat transfer coefficient
k	thermal conductivity of plate
N	number of data in sample
r	experimental result
s	standard deviation
t	time
P	uncertainty (95% confidence estimate)
q_w	heat flux from surface-to-fluid
T	temperature of plate
T_{aw}	adiabatic-wall temperature
T_i	initial temperature of plate
T_w	surface temperature of plate
X_i	measured variable
α	nondimensional heat transfer coefficient ($h\sqrt{t/\kappa}$)
β_i	derivative of r ($\partial r/\partial X_i$)
κ	thermal property of plate (ρck)
θ_{aw}	adiabatic temperature difference ($T_{aw} - T_i$)
θ_w	temperature difference ($T_w - T_i$)
Θ	nondimensional temperature (θ_w/θ_{aw})
ρ	density of plate
τ	penetration time
Φ	amplification parameter for uncertainties

Subscripts

h	with reference to heat transfer coefficient
min	minimum value
opt	optimum value
p	individual value
t	with reference to time
T	value for special case where uncertainties in measured temperatures are equal to each other
T_{aw}	with reference to adiabatic-wall temperature
T_i	with reference to initial temperature
T_w	with reference to surface temperature
α	with reference to α
θ_{aw}	with reference to θ_{aw}
Θ	with reference to Θ
κ	with reference to κ
1,2	values at times t_1, t_2

so-called semi-infinite-plate problem. This is the solution of the one-dimensional Fourier equation for the case of an infinitely thick plate, initially at a uniform temperature, T_i, exposed to a step-change in the temperature of the adjacent fluid. If T_{aw} is already known, then as shown in Section 2, it is only necessary to know the surface temperature, T_w, and the time, t, in order to evaluate h. For this case, a single narrow-band TLC provides the required value of T_w. If, however, T_{aw} is unknown, then two narrow-band TLCs, with respective transition temperatures of T_{w1} and T_{w2}, say, together with the associated times, t_1 and t_2, enable both h and T_{aw} to be evaluated, as shown in Section 3.

In practice, the test piece has a finite thickness, d say. The *penetration time*, τ, is defined as the time taken for the back surface of the plate to change by a given amount. For the case where the *front surface* has a step-change from T_i to T_w, then according to Schultz and Jones (1973)

$$\tau = 0.10 d^2 \frac{\rho c}{k}, \tag{1.2}$$

which corresponds to the time at which the change in the temperature of the *back surface* is 0.01 ($T_w - T_i$). For the case of a step-change in the *fluid temperature*, Eq. (1.2) provides a conservative estimate for τ. For experiments in which $t < \tau$, the semi-infinite assumption is taken to be valid.

Any uncertainties in the measured temperatures will obviously give rise to uncertainties in the calculated values of h. If T_{aw} is known, what is the value of T_w (which is fixed by the choice of TLC) that will minimise the uncertainty in h? If T_{aw} and h are both unknown, what are the values of T_{w1} and T_{w2} that will minimise their uncertainties? These are the questions addressed in this paper, and their answers should enable experimenters to quantify, and to minimise, the experimental uncertainties in h and T_{aw}.

Section 2 is concerned with the calculation of the uncertainty in h when T_{aw} is known, and Sections 3 and 4 respectively consider calculating and minimising the uncertainties when both h and T_{aw} are unknown. The conclusions are given in Section 5, and Appendix A contains the results of Coleman and Steele (1999) that are used in the uncertainty analysis presented below.

It should be noted that the results presented here are valid only for random uncertainties and it is implicitly assumed that there are no correlated biases between the temperatures. Coleman and Steele also discuss biases or systematic uncertainties, for which the methods used below should still be valid.

The analysis also implicitly assumes that h is invariant with time and it is therefore invalid for those problems in which h varies with either the magnitude or the distribution of surface temperature. In free convection, h depends on the magnitude of the difference between the temperatures of the surface and the fluid. In some transient forced convection problems, the changing thermal boundary conditions can affect the value of h (see Butler and Baughn, 1996). In neither of these cases is the analysis strictly valid, but the results may still be useful as a guide to the selection of TLC and the estimation of uncertainties in h.

2. Uncertainty in h when T_{aw} is known

For some heat transfer problems, the adiabatic-wall temperature is known before the experiment is undertaken. Under these conditions, if T_i, the initial temperature of the plate at time $t = 0$, and T_w, the surface temperature of the plate at time t, are known, then h can be readily found.

It is convenient to define

$$\Theta = \frac{\theta_w}{\theta_{aw}}, \tag{2.1}$$

where

$$\theta_w = T_w - T_i \tag{2.2}$$

and

$$\theta_{aw} = T_{aw} - T_i. \tag{2.3}$$

The solution of the semi-infinite-plate problem for a step-change in fluid temperature (see Schultz and Jones, 1973) can then be written as

$$\Theta = f(\alpha), \tag{2.4}$$

where

$$f(\alpha) = 1 - e^{\alpha^2}\,\mathrm{erfc}(\alpha) \tag{2.5}$$

and

$$\alpha = h\sqrt{\frac{t}{\kappa}}. \tag{2.6}$$

Knowing Θ from the temperature measurements, α can be calculated using Eq. (2.5); knowing t, h can then be found from Eq. (2.6).

As T_{aw} and T_i are usually fixed at the outset of the experiment, the choice of TLC used to measure T_w determines the value of Θ. As shown below, this also fixes the uncertainty in the measured values of h, but there is an optimum value of Θ that minimises this uncertainty.

If the random uncertainties in T_w, T_i and T_{aw} are independent of each other (as is usually the case in practice), the uncertainty in α can be found, with the aid of Appendix A, from

$$P_\alpha^2 = \left(\frac{d\alpha}{d\Theta}\right)^2 P_\Theta^2, \tag{2.7}$$

where

$$P_\Theta^2 = \left(\frac{\partial\Theta}{\partial T_w}\right)^2 P_{T_w}^2 + \left(\frac{\partial\Theta}{\partial T_i}\right)^2 P_{T_i}^2 + \left(\frac{\partial\Theta}{\partial T_{aw}}\right)^2 P_{T_{aw}}^2. \tag{2.8}$$

Using Eqs. (2.1)–(2.3), it follows that:

$$\left(\frac{P_\alpha}{\alpha}\right)^2 = \left(\frac{1}{\alpha f'(\alpha)}\right)^2 \left\{ \left(\frac{P_{T_w}}{\theta_{aw}}\right)^2 + (1 - f(\alpha))^2 \left(\frac{P_{T_i}}{\theta_{aw}}\right)^2 + f^2(\alpha)\left(\frac{P_{T_{aw}}}{\theta_{aw}}\right)^2 \right\}, \tag{2.9}$$

where

$$f'(\alpha) = \frac{df}{d\alpha} = 2[\alpha(f(\alpha) - 1) + \pi^{-1/2}]. \tag{2.10}$$

The uncertainty in h can then be found from Eq. (2.6) such that

$$\left(\frac{P_h}{h}\right)^2 = \left(\frac{P_\alpha}{\alpha}\right)^2 + \left(\frac{1}{2}\frac{P_t}{t}\right)^2 + \left(\frac{1}{2}\frac{P_\kappa}{\kappa}\right)^2. \tag{2.11}$$

Consider the special case where the uncertainties in t and κ are negligible, compared with P_h, and where $P_{T_w} = P_{T_i} = P_{T_{aw}} = P_T$, say. Eq. (2.9) then simplifies to

$$\left(\frac{P_h}{h}\right) = \Phi_h\left(\frac{P_T}{\theta_{aw}}\right), \tag{2.12}$$

where

$$\Phi_h = \frac{\{2(1 - f(\alpha) + f^2(\alpha))\}^{1/2}}{\alpha f'(\alpha)}. \tag{2.13}$$

Φ_h can be regarded as an *amplification parameter* that relates P_h/h to P_T/θ_{aw}, and the variation of Φ_h with Θ is shown in Fig. 1. Also shown are the computed values obtained using a Monte Carlo method in which random noise was added to T_w, T_i and T_{aw}, and the values of h were determined using Eq. (2.5). A total of $N = 10{,}000$ values was used to compute the average value of h and its uncertainty, P_h, and for the results shown in Fig. 1, $P_T = 0.2$ °C and $\theta_{aw} = 40$ °C, which are typical of the values used in experiments with TLC. The good agreement between the computed results and Eq. (2.12) gives confidence in the uncertainty analysis used here.

From Fig. 1, it can be seen that the minimum value of Φ_h ($\Phi_{h,\min} \approx 4.4$) occurs at an optimum value of Θ ($\Theta_{opt} \approx 0.52$). It is also interesting to note that $\Phi_h \leqslant 5$ for $0.3 < \Theta < 0.7$, which provides good latitude for the experimenter: for $P_T/\theta_{aw} = 0.5\%$ (the value chosen in the numerical simulation), $P_h/h \approx 2.5\%$ when $\Phi_h = 5$, which most experimenters would regard as acceptable.

It should be stressed that the above special case only applies if the uncertainties in the measured temperatures are all equal, which is not generally true. For other cases, Eq. (2.9) should be used to generate the appropriate amplification parameter. This would then allow Θ_{opt} and the appropriate value of P_h/h to be determined. (Good agreement between Eq. (2.9) and results obtained by the Monte Carlo method has also been found by the authors for cases where the uncertainties in temperature are unequal.)

3. Calculating the uncertainties in h and T_{aw}

If both h and T_{aw} are unknown before the experiment is conducted, they can be determined using two liquid crystals to measure temperatures T_{w1} and T_{w2}, say, at respective times of t_1 and t_2. Using the definition given in Eq. (2.1), these give rise to Θ_1 and Θ_2 such that

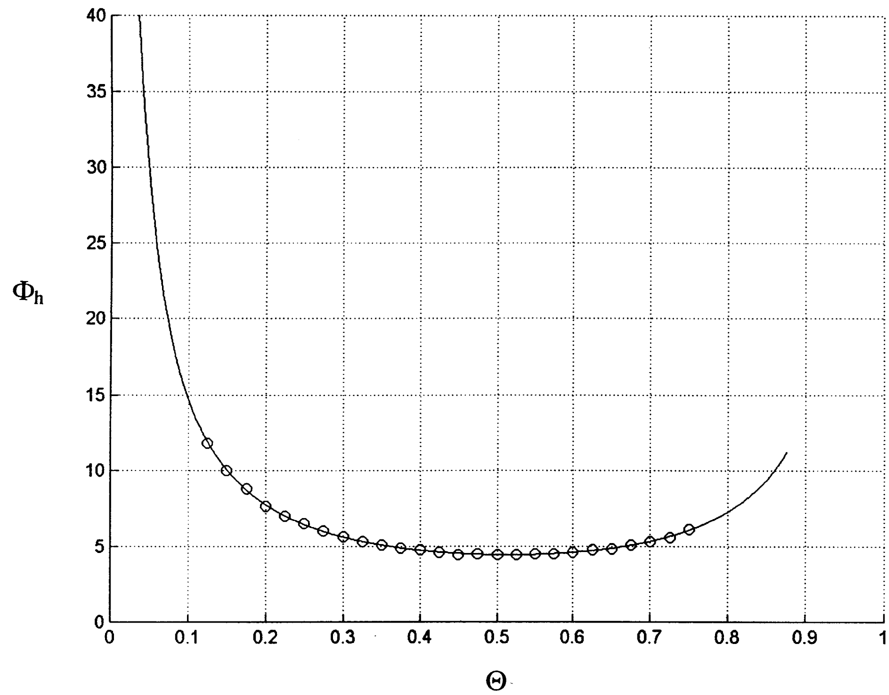

Fig. 1. Variation of Φ_h with Θ when T_{aw} is known. (———) Eq. (2.13); (○) computed values.

$$\frac{\Theta_1}{\Theta_2}=\frac{T_{w1}-T_i}{T_{w2}-T_i}. \tag{3.1}$$

Using Eq. (2.4),

$$\frac{\Theta_1}{\Theta_2}=\frac{f(\alpha_1)}{f(\alpha_2)}, \tag{3.2}$$

where

$$\alpha_1 = h\sqrt{\frac{t_1}{\kappa}} \tag{3.3}$$

and

$$\alpha_2 = h\sqrt{\frac{t_2}{\kappa}}. \tag{3.4}$$

Hence, as Θ_1 and Θ_2 are known, h and T_{aw} can be readily determined, and the uncertainties in h and T_{aw} can be calculated (see Yan and Owen, 2000). The relative uncertainties in h and T_{aw} can be expressed as

$$\left(\frac{P_h}{h}\right)^2 = \left(\Phi_1^{-1}-\Phi_2^{-1}\right)^{-2}\left\{\Theta_1^{-2}\left(\frac{P_{T_{w1}}}{\theta_{aw}}\right)^2+\Theta_2^{-2}\left(\frac{P_{T_{w2}}}{\theta_{aw}}\right)^2 + \left(\Theta_1^{-1}-\Theta_2^{-1}\right)^2\left(\frac{P_{T_i}}{\theta_{aw}}\right)^2\right\} \tag{3.5}$$

and

$$\left(\frac{P_{T_{aw}}}{\theta_{aw}}\right)^2 = (\Phi_2-\Phi_1)^{-2}\left\{\Phi_1^2\Theta_1^{-2}\left(\frac{P_{T_{w1}}}{\theta_{aw}}\right)^2 + \Phi_2^2\Theta_2^{-2}\left(\frac{P_{T_{w2}}}{\theta_{aw}}\right)^2 + \left[\Phi_2\left(\Theta_2^{-1}-1\right) - \Phi_1\left(\Theta_1^{-1}-1\right)\right]^2\left(\frac{P_{T_i}}{\theta_{aw}}\right)^2\right\}, \tag{3.6}$$

where

$$\Phi_1 = \left(\frac{f(\alpha)}{\alpha f'(\alpha)}\right)_1 \tag{3.7a}$$

and

$$\Phi_2 = \left(\frac{f(\alpha)}{\alpha f'(\alpha)}\right)_2. \tag{3.7b}$$

The minimisation of these uncertainties is discussed below.

4. Minimising the uncertainties in *h* and T_{aw}

For simplicity, consider the special case where $P_{T_{w1}} = P_{T_{w2}} = P_{T_i} = P_T$, say. Eqs. (3.5) and (3.6) then reduce to

$$\frac{P_h}{h} = \Phi_h\frac{P_T}{\Theta_{aw}}, \tag{4.1a}$$

where

$$\Phi_h = \left\{2\left(\Phi_1^{-1}-\Phi_2^{-1}\right)^{-2}\left(\Theta_1^{-2}+\Theta_2^{-2}-\Theta_1^{-1}\Theta_2^{-1}\right)\right\}^{1/2} \tag{4.1b}$$

and

$$\frac{P_{T_{aw}}}{\Theta_{aw}} = \Phi_{T_{aw}}\frac{P_T}{\Theta_{aw}}, \tag{4.2a}$$

where

$$\Phi_{T_{aw}} = |\Phi_2 - \Phi_1|^{-1}\Big\{\Phi_1^2\Theta_1^{-2} + \Phi_2^2\Theta_2^{-2} + \big[\Phi_2(\Theta_2^{-1} - 1) - \Phi_1(\Theta_1^{-1} - 1)\big]^2\Big\}^{1/2}. \quad (4.2b)$$

Φ_h and $\Phi_{T_{aw}}$ are *amplification parameters*: they respectively relate the uncertainties in the derived quantities (P_h/h and $P_{T_{aw}}/\theta_{aw}$) to the uncertainties in the measured temperatures (P_T/θ_{aw}).

Fig. 2 shows the effect of Θ_2 on the variation of P_h/h with Θ_1 according to Eqs. (4.1a) and (4.1b). Computed values, obtained using the Monte Carlo method described in Section 2, are also shown, and the good agreement between Eqs. (4.1a) and (4.1b) and the computations gives confidence in the uncertainty analysis used here.

It can be seen from Fig. 2 that P_h/h tends to decrease as Θ_2 increases, and for any value of Θ_2 there is an optimum value of Θ_1 ($\Theta_1 = \Theta_{1,opt}$, say) for which P_h/h is a minimum. The locus of the minima is also shown, and

$$\Theta_{1,opt} \approx 0.52\Theta_2. \quad (4.3)$$

It is also interesting to note that, as Θ_2 tends to unity, the variation of P_h/h with Θ_1 is similar to that of P_h/h with Θ shown in Fig. 1. That is, Eq. (2.12) provides a lower bound for the amplification parameter, and $\Phi_{h,min} \approx 4.4$. A poor choice of Θ_1 and Θ_2 can, however, result in values of Φ_h an order of magnitude greater than this minimum value, as Fig. 2 shows. The danger for the unwary experimenter is clear to see!

Fig. 3 shows the effect of Θ_2 on the variation of $P_{T_{aw}}/T_{aw}$ with Θ_1 according to Eq. (4.2a) and (4.2b), and the agreement between the results from this equation and the computations is good. As for P_h/h, $P_{T_{aw}}/\theta_{aw}$ tends to decrease as Θ_2 increases, and there is an optimum value for which $P_{T_{aw}}/\theta_{aw}$ is a minimum. This optimum value can be approximated by

$$\Theta_{1,opt} \approx 0.48\Theta_2. \quad (4.4)$$

It can also be seen from Figs. 2 and 3 that the minimum values of P_h/h are significantly greater than those of $P_{T_{aw}}/\theta_{aw}$.

For experimenters, the best strategy is to choose TLC with transition temperatures that make Θ_2 as large as praticable and make $\Theta_1 \approx 0.5\Theta_2$. However, the maximum value of Θ_2 may be limited in practice by the need to ensure that the experimental time, t, does not exceed the penetration time, τ (see Section 1).

In order to choose Θ_1 and Θ_2 before the experiment is conducted, it is necessary to have an estimate of the unknown T_{aw}. In practice, T_{aw} will be related to the total temperature of the fluid entering the system and, in most experiments, it is possible to control and to measure this temperature. The magnitude of the uncertainties is therefore in the gift of the experimenter: a well-designed experiment will minimise these uncertainties.

It should be remembered that the results presented in this section are only valid when $P_{T_{w1}} = P_{T_{w2}} = P_{T_i}$. When this is not the case, Eqs. (3.5) and (3.6) can be used to produce results similar to those shown in Figs. 2 and 3.

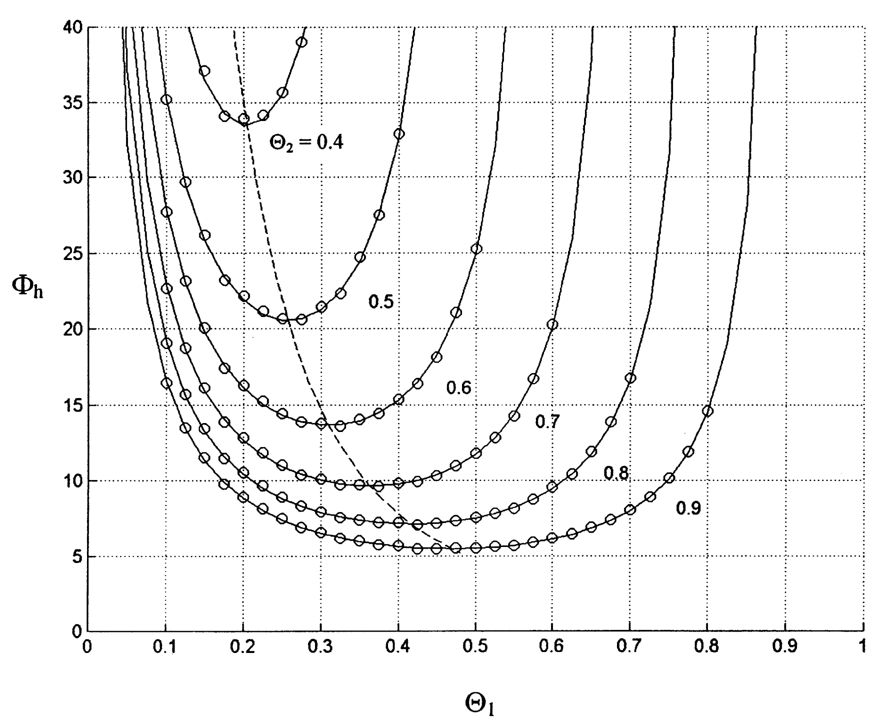

Fig. 2. Effect of Θ_2 on variation of Φ_h with Θ_1 when T_{aw} is unknown. (———) Eqs. (4.1a) and (4.1b); (- - -) locus of minima; (○) computed values.

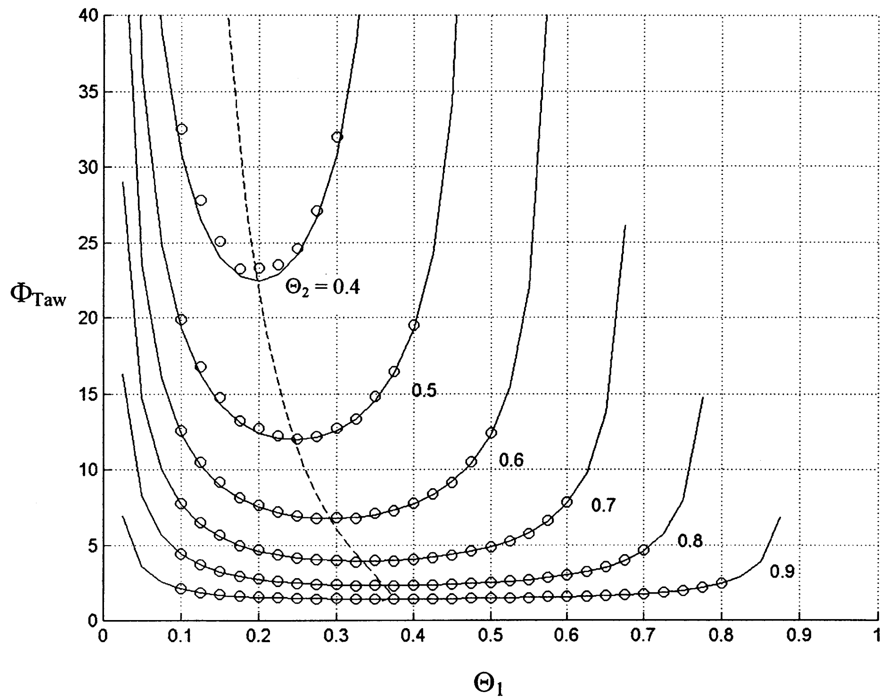

Fig. 3. Effect of Θ_2 on variation of $\Phi_{T_{aw}}$ with Θ_1. (———) Eq. (4.2a) and (4.2b); (- - -) locus of minima; (○) computed values.

5. Conclusions

Using the step-change solution of Fourier's equation for a semi-infinite plate, analytical expressions have been derived for P_h, the uncertainty in h, and for $P_{T_{aw}}$, the uncertainty in T_{aw} (when T_{aw} is unknown), in terms of the random uncertainties in the measured temperatures. These expressions are in good agreement with computed values obtained using a Monte Carlo method.

When T_{aw} is known, there is an optimum value of the nondimensional temperature, Θ, that minimises P_h. For the special case where the uncertainties in the measured temperatures, P_T, are equal to each other, $\Theta_{opt} \approx 0.5$. For this case, the amplification parameter (or ratio of P_h/h to P_T/θ_{aw}) is approximately 4.4.

When T_{aw} is unknown, two values of Θ (Θ_1 and Θ_2) are needed to determine h and T_{aw}. For any value of Θ_2, there is an optimum value of Θ_1 that minimises the uncertainty in h. For the special case where the uncertainties in the measured temperatures are equal, $\Theta_{1,opt} \approx 0.5\Theta_2$, and P_h/h and $P_{T_{aw}}/\theta_{aw}$ decrease as Θ_2 increases. The advice to experimenters is to make Θ_2 as large as practicable and to choose the optimum value of Θ_1 to minimise P_h; a poor choice of Θ_1 and Θ_2 could result in very large uncertainties in P_h/h and $P_{T_{aw}}/\theta_{aw}$.

Although the results presented here are valid only for random uncertainties in the measured temperatures, Coleman and Steele (1999) provide formulae for biases or systematic uncertainties. It should therefore be possible to use the methods in this paper to determine the uncertainties in h and T_{aw} resulting from biases in the measured temperatures.

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Appendix A. Estimating random uncertainties

The following has been extracted from Coleman and Steele (1999) for the case of large samples of data ($N \geqslant 10$).

Consider the case of an experimental result, r, which is a function of J measured variables, X_i such that

$$r = r(X_1, X_2, \ldots, X_J). \tag{A.1}$$

The random uncertainty (precision limit) of the result is given by

$$P_r^2 = \sum_{i=1}^{J} \beta_i^2 (P_i)^2 + 2 \sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \beta_i \beta_k P_{ik}, \tag{A.2}$$

where

$$\beta_i = \frac{\partial r}{\partial X_i} \tag{A.3}$$

P_i is the 95% confidence estimate of the random uncertainty in X_i, and is given by

$$P_i^2 = 4S_i^2, \tag{A.4}$$

where the variance, S_i^2, is found from

$$S_i^2 = \frac{1}{N-1}\sum_{p=1}^{N}\left(X_{i,p} - \overline{X}_i\right)^2 \tag{A.5}$$

and $\overline{X}_i$ is the mean value of the N samples of X_i.

P_{ik} is the 95% confidence estimate of the covariance of X_i and X_k given by

$$P_{ik} = 4S_{ik}, \tag{A.6}$$

where

$$S_{ik} = \frac{1}{N-1}\sum_{p=1}^{N}\left(X_{i,p} - \overline{X}_i\right)\left(X_{k,p} - \overline{X}_k\right). \tag{A.7}$$

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